Although calculations of this type do not predict the proper behavior for the ω-β characteristic near stopbands resulting from a periodic perturbation, they do predict the occurrence and width of such stop bands.

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Ice as a Bending Medium for Waveguide and Tubing*

Bending waveguide and metal tubing is very often a difficult and time-consuming task. Low melting temperature alloys are at times difficult to remove from waveguide and tubing. The piece to be bent may be filled with water which is then frozen by dry ice, liquid nitrogen, or by a deep freeze. In some applications where the piece to be bent is integral with a larger system, a block of dry ice may be held against it to freeze only the portion of water around the section to be bent. The use of these low temperatures causes not only the water to freeze into quite small crystals (which act like a sand packing), but also prevents the ice from melting because of the pressure of bending.

Several tests were performed on thin walled aluminum tubing and P-band brass waveguide. It was found that in comparison to low melting alloys the bends were identical within the statistical variation of samples. The time required for the operation was considerably shorter.

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On Higher-Order Hybrid Modes of Dielectric Cylinders*

In the course of investigations into the properties of various surface wave structures,[1] it became necessary to investigate hybrid modes on dielectric cylinders for modes of order n, where $n>1$. The case $n=1$ has received extensive treatment in the literature [1]–[6].

The radial dependence of the axial fields is as $J_n[p(\rho/a)]$ inside the dielectric cylinder and $K_n[q(\rho/a)]$ outside, where ρ is the radial cylindrical coordinate, a is the radius of the cylinder, p and q are radial eigenvalues, and n is the rank of the mode.

The requirement of continuity of the fields at the boundary leads, in the usual manner, to the characteristic equation involving Bessel functions and their derivatives. This was first given by Schelkunoff [4]. The derivatives of Bessel functions may be eliminated from this equation by the use of identities such as given by Watson [8], to yield the simple form

$$(J^+ + K^+)(\epsilon J^- - K^-) + (J^- - K^-)(\epsilon J^+ + K^+) = 0, \quad (1)$$

where

$$J^- = \frac{J_{n-1}(p)}{pJ_n(p)}, \qquad J^+ = \frac{J_{n+1}(p)}{pJ_n(p)};$$

$$K^- = \frac{K_{n-1}(q)}{qK_n(q)}, \qquad K^+ = \frac{K_{n+1}(q)}{qK_n(q)};$$

and ϵ is permittivity of dielectric cylinder relative to surrounding medium.

The cutoff values of the parameter p are of great interest; they may be obtained by letting $q \to 0$ in the characteristic equation. To keep the terms finite requires that the equation be multiplied by an appropriate power of q before the limit is taken. If it is assumed that J^- is finite at cutoff, it is sufficient to multiply the equation by q^2 to obtain a solution for the cutoff values of p; this was given by Schelkunoff [4]. However, if this assumption is not made, an additional solution may be determined. This will be outlined below.

Multiplying the characteristic equation by $[qpJ_n(p)]^2$ gives

$$(q^2J_{n+1} + q^2K^+pJ_n)(\epsilon J_{n-1} - pJ_nK^-) + (J_{n-1} - pJ_nK^-)(\epsilon q^2J_{n+1} + q^2K^+pJ_n) = 0. \quad (2)$$

Taking the limit as $q \to 0$ and noting that

$$K^- \to \frac{1}{2(n-1)}$$

and $q^2K^+ \to 2n$ one obtains

$$2n\, pJ_n\left((\epsilon+1)J_{n-1} - \frac{pJ_n}{n-1}\right) = 0. \quad (3)$$

The solutions are, for $n>1$,

$$\frac{J_{n-1}(p)}{pJ_n(p)} = J^- = \frac{1}{(n-1)(\epsilon+1)}, \quad (4)$$

$$J_n(p) = 0, \qquad p \neq 0. \quad (5)$$

Eq. 4 is given by Schelkunoff [4]. The very significant exclusion of the $p=0$ solution of (5) as a cutoff condition is based on the fact that for $q\to 0$ and $p \to 0$, (1) becomes, since

$$J^- \to \frac{2n}{p^2}, \qquad J^+ \to \frac{1}{2(n+1)},$$

$$\left(\frac{1}{2(n+1)}+\frac{2n}{q^2}\right)\left(\frac{2n\epsilon}{p^2}-\frac{1}{2(n-1)}\right) + \left(\frac{2n}{p^2}-\frac{1}{2(n-1)}\right)\left(\frac{\epsilon}{2(n+1)}+\frac{2n}{q^2}\right) = 0. \quad (6)$$

When the finite terms are neglected in comparison with the infinite terms, it is seen that this is not satisfied at $q=0$, $p=0$ for any $n>1$. However, the $p=q=0$ solution, *i.e.*, the condition for "no cutoff," is valid for $n=1$ [1].

The asymptotes for the p-q curves are of interest. For $q\to\infty$ the characteristic equation becomes simply $2\epsilon J^-J^+=0$, with solutions at $J_{n-1}(p)=0$ and $J_{n+1}(p)=0$. It will be seen that the first of these is associated with the modes satisfying the first or Schelkunoff cutoff condition, the second with the alternate cutoff condition given here in (5).

Because of the oscillatory character of $J_n(p)$, the characteristic equation is satisfied by an infinite set of values of p for any given q, in particular also for $q=0$. These sets of p's span an infinite set of modes which may propagate along the dielectric rod. It is now seen that the existence of the alternate cutoff condition indicates the existence of an infinite set of modes that interlace the modes that satisfy the cutoff condition of (4). This and other salient characteristics of the doubly infinite set of modes are presented qualitatively in Fig. 1, with the $n=1$ case treated by Beam [1] included for comparison in Fig. 2. The curve shapes are based upon the detailed numerical solution of (2) obtained with an IBM 650 computer for $n=2$, 6 for a wide range of ϵ.

The significance of Fig. 1 may be summarized as follows.

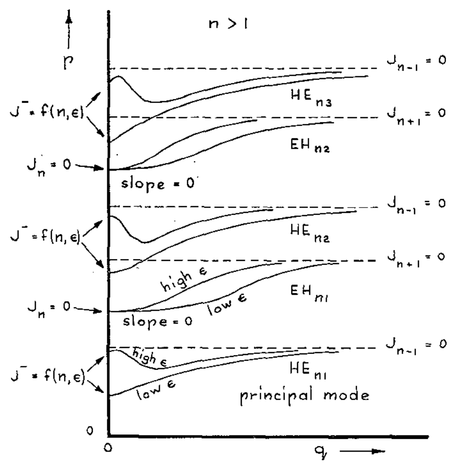

Fig. 1—Loci of solutions of the characteristic equation (1) for $n>1$.

$$f(n,\epsilon) = \frac{1}{(n-1)(\epsilon+1)}.$$

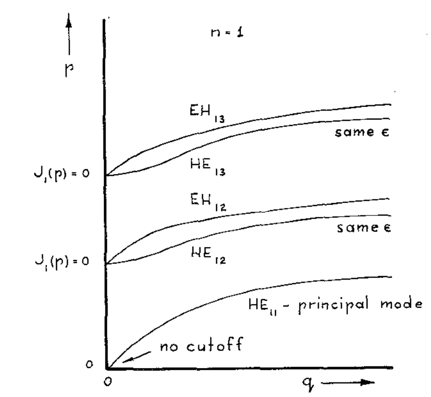

Fig. 2—Curves of p and q for $n=1$.

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[1] Report in preparation.

1) There is a set of modes that alternates with those identified with the cutoff condition of Schelkunoff in (4). Their cutoff condition is given by (5).

2) The modes identified with (4) have a cutoff that depends on ϵ, whereas the modes satisfying the alternate condition have a single cutoff value of p for all ϵ.

3) For $n>1$ all modes have some cutoff value. The principal mode for $n=1$ has no cutoff. There is no "degeneracy" of the modes for $n>1$ as there is for $n=1$ modes, as described by Beam [1]; that is, each mode has its own distinct cutoff point.

4) There is a unique principal mode for all n.

5) The existence of an additional mode between successive Schelkunoff modes reduces the upper frequency limit at which a pure principal mode may propagate below what the limit would be if only Schelkunoff modes existed, as follows.

The p–q curves may be used to determine the frequency dependence of the mode propagation with the aid of the additional relation

$$p^2 + q^2 = R^2 \tag{7}$$

where $R=(2a/\lambda_0)\pi\sqrt{\epsilon-1}$. This indicates that for a given dielectric rod of radius a, the actual values of p and q may be found at the intersection of the p–q curves with a superimposed circle of radius R corresponding to the frequency of operation for which the free-space wavelength is λ_0. To insure the propagation of a unique mode, the circle must intersect the p–q curves only once. For the principal mode, this means that the upper limit of R, and hence of frequency, is determined by the requirement that $R<p_0$, where $J_n(p_0)=0$. The lower limit of frequency is of course determined by the cutoff value of p (see Fig. 3).

It is now seen that there is no degeneracy to impose a notational distinction, so that the modes could be simply numbered successively. In the interest of conforming to

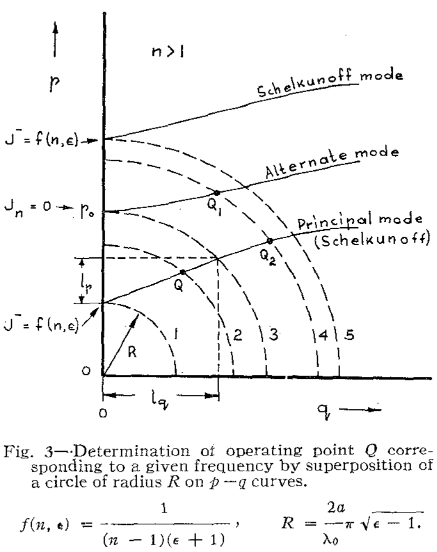

Fig. 3—Determination of operating point Q corresponding to a given frequency by superposition of a circle of radius R on p–q curves.

$$f(n, \epsilon) = \frac{1}{(n-1)(\epsilon+1)}, \qquad R = \frac{2a}{\lambda_0}\pi\sqrt{\epsilon-1}.$$

1) Lower limit of R. 2) Typical R. 3) Upper limit of R. 4) $R>p_0$. 5) Upper limit of R in absence of alternate modes. Q: operating point for typical R. Q_1, Q_2: two operating points for $R>p_0$: impure mode. l_p, l_q: useful ranges of p, q, for pure principal mode.

the nomenclature for $n=1$, however, and in order to preserve the distinction between the modes that satisfy the Schelkunoff cutoff condition and those that satisfy the alternate condition, the HE_{nm}, EH_{nm} distinction is retained here, starting with HE_{n1} for the principal mode.

An attempt to verify a possible distinction between H- and E-type modes for the general case of any n, as suggested by Wegener and others [1], [7] for $n=1$, has not been found by the authors to lead to consistent results. The designation here of a mode as HE_{nm} is hence not to be construed as an indication that the mode must be H type.

The existence of the alternate cutoff condition, (5), has been confirmed independently using approximation methods by Snitzer [9] in the course of his investigation into the optical properties of thin fibers.

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A Property of Symmetric Hybrid Waveguide Junctions*

It is well known that in symmetric hybrid junctions such as the short-slot, branched-guide, and transvar types, the signals in the main and auxiliary guides are in phase quadrature. Another property of fully symmetric lossless hybrids is that if all the arms are matched, the amplitudes of the waves traveling in the reverse direction in the main and auxiliary guides are equal. A proof is given below.

Since in a well-designed hybrid these amplitudes will be of the order of 0.03 or less, relative to the input, this is not an easy fact to observe experimentally. However, if the measured VSWR and isolation of such a hybrid are inconsistent, one may reflect that this must be because of

1) experimental error,
2) mismatch of terminations or bends introduced for purposes of measurement,
3) asymmetry allowed by manufacturing tolerances,
4) ohmic loss.

Proof: Let arms 1–3 be the main guide, and arms 2–4 the auxiliary guide. If the hybrid is fully symmetric its scattering matrix will have the form,

$$S = \begin{pmatrix} A & B & C & D \\ B & A & D & C \\ C & D & A & B \\ D & C & B & A \end{pmatrix},$$

where A and B are small, and C and D have approximately equal amplitude. If the hybrid is lossless, S is unitary, which gives us

$$\mathrm{Re}\,(A\bar{B}) + \mathrm{Re}\,(C\bar{D}) = 0, \tag{1}$$

$$\mathrm{Re}\,(A\bar{C}) + \mathrm{Re}\,(B\bar{D}) = 0, \tag{2}$$

$$\mathrm{Re}\,(A\bar{D}) + \mathrm{Re}\,(B\bar{C}) = 0, \tag{3}$$

where the bar denotes the complex conjugate. Let $A=A_1+jA_2$, and similarly for B and D, and let the reference planes be chosen so that C is real. Then (1) shows that D_1 is a second-order small quantity, leading to the first property that C and D are in quadrature.

Putting $D=jC$, we have from (2) and (3), respectively,

$$A_1 = -B_2,$$

$$A_2 = -B_1.$$

Hence $A=-j\bar{B}$, and A and B have the same amplitude.

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Attenuation in a Resonant Ring Circuit*

The use of a resonant circuit will permit raising the strength of electromagnetic fields to values considerably higher than that available directly from a transmitter. In the usual resonant cavity, standing waves exist which may raise some doubt as to the usefulness of this method for testing certain

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